18.600 Recitation 10

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Partial solutions available at math.mit.edu/∼visheshj

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Problem 1. Define X as the height in meters of a randomly selected Canadian, (where the selection probability is equal for each Canadian), and denote $E[X]$ by h. Bo is interested in estimating h. Because he is sure that no Canadian is taller than 3 meters, Bo decides to use 1.5 meters as a conservative (large) value for the standard deviation of X. To estimate h, Bo averages the heights of n Canadians that he selects at random; he denotes this quantity by H.

(a) In terms of h and Bos 1.5 meter bound for the standard deviation of X, determine the expectation and standard deviation for H.

(b) Help Bo by calculating a minimum value of n (with $n > 0$) such that the standard deviation of Bos estimator, H, will be less than 0.01 meters.

(c) Say Bo would like to be 99% sure that his estimate is within 5 centimeters of the true average height of Canadians. Using the Chebyshev inequality, calculate the minimum value of n that will make Bo happy.

(d) If we agree that no Canadians are taller than three meters, why is it correct to use 1.5 meters as an upper bound on the standard deviation of X, the height of any Canadian selected at random?

Problem 2. Let X be a continuous random variable with pdf $f_X(x) = \frac{e^{-|x|}}{2}$, for all $-\infty < x < \infty$.

(a) Calculate the moment generating function of X, and $E[X]$, $Var(X)$.

(b) Calculate the moment generating function of N, the normal random variable of mean $E[X]$ and variance $Var(X)$. Denote this by $M_N(t)$.

(c) Let $X_1, X_2, \dots$ be independent instances of X. For any $n \in \{1, 2, \dots\}$, calculate the moment generating function of $S_n = \frac{X_1 + \cdots + X_n}{\sqrt{n}}$ and compare the limit of the sequence $M_{S_n}(t)$ (as n tends to infinity) with $M_N(t)$.

Problem 3. Is the Chebyshev inequality tight? That is, for every $\mu, \sigma \geq 0$, and $c \geq \sigma$, does there exist a random variable X with mean μ and standard deviation σ such that

$$\Pr(|X - \mu| \geq c) = \frac{\sigma^2}{c^2}?$$