18.600 Recitation 11

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Partial solutions available at math.mit.edu/∼visheshj

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Problem 1. Assume MIT has 4000 undergrads, and that in a given semester, each student enrolls in the introductory probability class with probability 5% (with the decision to enroll in the class taken independently for each student). The professor in charge of the course has decided that if the number enrolling is 250 or more, he will teach the course in two separate sections, whereas if fewer than 250 students enroll, he will teach all of the students together in a single section.

(a) By using normal approximation, find the approximate probability that the professor will be teaching two sections.

(b) Find a bound for the probability that the professor will be teaching two sections using Markov's inequality.

(c) Repeat (b) using a Chernoff bound with $t = 0.2$. You may use that the moment-generating function for a binomial random variable X with parameters n and p is given by $M_X(t) = (pe^t - p + 1)^n$.

Problem 2. Consider a frog and four boxes, such that the boxes are placed in a circle labelled from 1 to 4 in counterclockwise direction. Define a Markov chain $X_0, X_1, \ldots$ as follows for some $p \in [0,1]$. At each step in time the frog jumps to one of the two adjacent boxes. The frog jumps in counterclockwise direction with probability p and in clockwise direction with probability $1-p$. For example, if the frog is in box 1 it jumps to box 2 with probability $1-p$ and to box 4 with probability p. Let $X_k \in \{1,2,3,4\}$ represent the box containing the frog at time k.

(a) Assuming $X_0 = 1$, what is the distribution of X_n?

(b) Is the Markov chain ergodic?

Now modify the transition rules of the Markov chain: Assume that at each point in time, the frog does one of the following with equal probability: (1) It stays in the same box, (2) it jumps in counterclockwise direction, or (3) it jumps in clockwise direction.

(c) What is the distribution of X_k when k goes to infinity?