18.600 Recitation 3

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Problem 1. Make four fair coin tosses, and let X be the number of heads. Let $Y = X(X-2)^2(X-4)$? What is $\mathbb{E}[Y]$? What is $\mathbb{E}[Y^4]$? Is it true that $\mathbb{E}[Y^4] = \mathbb{E}[Y]^4$?

Problem 2. A delighted fan at a basketball game is selected for a free-throw contest at half-time. He is allowed to take three shots, and he will be given $500 for each one he successfully makes. Assume he successfully makes the first two shots independently with probability 0.2 each, and the last one independently with probability 0.5 (since he is more focused then!) What are his expected winnings?

Problem 3. Two fair dice are rolled. Let X equal the sum of the two dice.
(i) Find the expectation of X using the probability mass function.
(ii) Find the expectation of X using linearity of expectation.
(iii) Find the expectation of X^2.

Problem 4. Consider a box with n apples, b of which have gone bad. We choose k apples at random from the box.
(i) What is the expected number of bad apples we choose, provided the apples are chosen *with replacement*?
(ii) What is the expected number of bad apples we choose, provided the apples are chosen *without replacement*?

Optional Problem. *The St. Petersburg paradox.* You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars.
(i)What is the expected amount that you will receive?
(ii) Find the cumulative distribution function $F_W(x)$ for your winnings.