

18.600 Recitation 4

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Problem 1. A chocolate factory produces four kinds of products: white chocolate, milk chocolate, dark chocolate, and caramel. They sell bags consisting of 10 items, where each item is chosen independently, and is equally likely to be any one of the above four kinds.

(a) What is the probability that a random bag contains exactly 3 pieces of milk chocolate?

(b) If a bag does not contain at least two pieces of caramel and at least two pieces of chocolate (the kind of chocolate doesn't matter), it is considered to be defective. What is the probability that a random bag is defective?

Problem 2. Alice and Bob play a series of games against each other. Alice wins each game with probability $2/3$, and Bob wins each game with probability $1/3$. Let X be the time it takes until Bob wins for the first time. Find $P[X > 5]$ and $E[X]$.

Problem 3. Consider the following two random variables X and Y :

- Throw a die until you get a six, and let X be the number of times you had to throw it.
- A soccer player practices penalty kicks. She shoots at the goal four times, and succeeds with probability $1/3$ at each shot. Let Y be the number of times she scores.

Define $Z = X + Y$.

(a) What is $E(Z)$?

(b) What is $P(Z = 2)$?

(c) If F_Z is the cumulative distribution function of Z , what is $F_Z(2)$?

Problem 4. Two teams Team A and Team B are playing against each other in a quiz with 10 questions. Each question is answered correctly by Team A with probability $3/4$, independently of their results on the other questions. Each question is answered correctly by Team B with probability $2/3$, independently of their results on the other questions and the results of Team A.

(a) Let X be the number of questions that are answered correctly by both teams. Find $E[X]$.

(b) Let Y be the number of questions that are answered correctly by at least one of the teams. Find $\text{Var}[Y]$.

(c) Find $E[Y^2]$. (Hint: Use the formula $\text{Var}[Y] = E[Y^2] - E[Y]^2$)