# 18.600 Recitation 5

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**Problem 1.** An urn contains $r$ red balls and $b$ blue balls, and balls are randomly drawn one at a time from it without replacement.

(a) Suppose you draw 3 balls from the urn. What is the probability that you draw more red balls than blue balls?

(b) Suppose that the red balls are labeled $R_1, \ldots, R_r$, and let $E_i$ be the event that the ball $R_i$ is drawn before the first blue ball. What is the probability of $E_i$?

(c) What is the expected number of red balls drawn before the first blue ball?
**Hint:** You may find it useful to express this using indicator variables for the $E_i$.

**Problem 2.** A system has 1000 components. Each component works with probability 0.998 and fails with probability 0.002, independently of all other components.

(a) Let $X$ be the number of components that work. Find the expectation and variance of $X$.

(b) The system works if at most 2 components fail. What is the probability that the system works?

(c) What is the conditional probability that component 1 works given that the system works?

(d) Find an approximation to the probability that the first 500 components all work, and exactly two of the second five hundred components fail (its okay for your answer to have $e$ in it, but no factorials or binomial coefficients).

**Problem 3.** Three fair dice, painted red, green and blue, are thrown. Assume that the outcomes of the three dice are independent, and consider the following events:

- Let $R$ be the event that the red die lands on an odd number.
- Let $G$ be the event that the green die lands on an odd number.
- Let $B$ be the event that the blue die lands on an odd number.

(a) Let $E$ be the event that the sum of the three dice is odd. Express the event $E$ in terms of the events $R$, $G$, $B$, and their complements.

(b) Let $X$ be the number of dice that land on an odd number. Find the cumulative distribution function of $X$.

(c) Compute $\mathrm{E}[X]$ (for $X$ defined as above).

**Problem 4.** Suppose that when Alice drives to work:

- There are 10 traffic lights along the way.

- Each traffic light is red with probability 1/4.
- The total time $X$ that it takes her is 15 minutes plus 2 minutes for each red light.

(a) What is the probability mass function of $X$?

(b) Compute $\mathrm{E}[X]$ and $\mathrm{Var}(X)$.

(c) Walking to work always takes Alice 20 minutes. Suppose that Alice drives to work with probability 1/3 and walks with probability 2/3, and let $Y$ the amount of time it takes her. What is the probability mass function of $Y$ ?