18.600 Recitation 6

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Problem 1. On weekends, Superman likes to fly home from Metropolis to Smallville, Kansas to visit his mother, a distance of 1000 miles. Suppose that Superman's flight speed S is distributed as $\max\{0, X\}$, where X is normally distributed with mean $\mu = 500$mph and standard deviation $\sigma = 100$mph. (Assume his speed is constant during a given flight, but it varies from weekend to weekend.) Find the:

(a) Probability he gets to Smallville in less than 1.5 hours.
(b) PDF of the time T (in hours) it takes Superman to get home.

Problem 2. A system has 1000 components. Each component works with probability 0.998 and fails with probability 0.002, independently of all other components. Find an approximation to the probability that the first 500 components all work, and exactly two of the second five hundred components fail (its okay for your answer to have e in it, but no factorials or binomial coefficients).

Problem 3. Suppose that earthquakes occur in the western portion of the United States with the following distribution: in any given time interval of length t weeks, the number of earthquakes which occur is a Poisson random variable with parameter $2t$.

(a) Find the probability that at least 3 earthquakes occur during the next 2 weeks.

(b) Find the cumulative distribution function of the time, starting from now, until the next earthquake.

Problem 4. Define a continuous random variable X by the following distribution:

$$f_X(x) = \begin{cases} \frac{x}{2} & \text{if } 0 \le x \le 2 \\ 0 & \text{else,} \end{cases}$$

and let $Y = e^{X^2}$. Find: (a) $E[Y]$, (b)$\mathrm{Var}[Y]$, (c) the CDF of Y, (d) the PDF of Y.

Problem 5. Mensa is an organization open to people with IQs in the top 2% of the population. If IQs are normally distributed with a mean of 100 and a standard deviation of 15, what is the minimum IQ necessary for admission?

Problem 6. Two points are placed uniformly at random on a line segment of length 1, breaking it up into three smaller segments. What is the probability that these three segments can form a triangle?