

18.600 Recitation 8

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Partial solutions available at math.mit.edu/~visheshj

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Problem 1. Let the random variables X and Y have a joint PDF which is uniform over the triangle with vertices $(0, 0)$, $(0, 1)$, and $(1, 0)$.

- (a) Find the joint PDF of X and Y .
- (b) Find the marginal PDF of Y .
- (c) Find the conditional PDF of X given Y .
- (d) Find $\mathbb{E}[X|Y = y]$, and use the total expectation theorem to find $\mathbb{E}[X]$ in terms of $\mathbb{E}[Y]$.
- (e) Use the symmetry of the problem to find the value of $\mathbb{E}[X]$.

Problem 2. A deck of 52 cards is shuffled and you are dealt a hand of 13 cards. Let X and Y denote, respectively, the number of aces and the number of spades in your hand.

- (a) Show that X and Y are uncorrelated.
- (b) Are they independent?

Problem 3. If N is a geometric random variable with parameter p , find $\mathbb{E}[\cos(N\pi)]$.

Problem 4. (*Elchanan Mossel's Amazing Dice Paradox*) You throw a die until you get 6. What is the expected number of throws (including the throw giving 6) conditioned on the event that all preceding throws gave even numbers? What is the probability that the first die is a 6 conditioned on the event that all preceding throws gave even numbers? Is this consistent with your answer for the expectation?