18.600 Recitation 9

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Partial solutions available at math.mit.edu/∼visheshj

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Problem 1. Let A and B be two random variables. Is it *always* true that $E[|A+B|] = E[|A|] + E[|B|]$? Why, or why not? What if A and B are independent random variables?

Problem 2. Let X, Y be independent exponential random variables with parameter 1. Let $V = \max\{X, Y\}$. Find the conditional expectation of V given that X=1.

Problem 3. Let (X, Y) be uniformly distributed on the unit disc.

(a) What is the joint PDF of X and Y?

(b) Find $E[X]$.

(c) Find $E[Y^3]$.

(d) Find $E[X^8 - Y^8]$.

Problem 4. Let X and Y be independent random variables such that:

- X is a normal random variable with mean 1 and standard deviation 3, and
- Y is a normal random variable with mean 5 and standard deviation 2.

(a) Find the PDF of $3X - 6Y$.

(b) Find the PDF of $|2X + 1|$.

(c) Find the PDF of $\max\{X, Y\}$.

Problem 5. Let X and Y be random variables generated as follows:

- X is chosen according to the uniform distribution on $[0, 1]$.
- Given the value of X, Y is then chosen according to the uniform distribution on $[-X, X]$.

(a) Are X and Y independent?

(b) Find $E[Y]$.

(c) Find $\mathrm{Var}[Y]$.

Problem 6. (a) Give an example of uncorrelated random variables X and Y which are not independent.

(b) Suppose X and Y are uncorrelated *indicator* random variables. Are X and Y independent?

(c) What is $\mathrm{Cov}(aX + b, cY + d)$ in terms of $\mathrm{Cov}(X, Y)$?